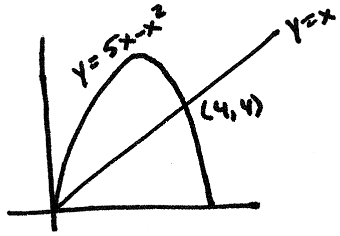
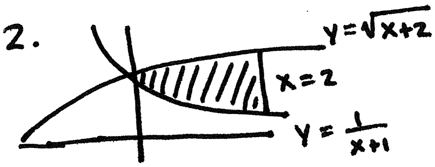
hw set 13
sect 6.1
p446

1.



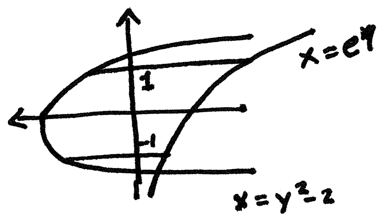
$$A = \int_0^4 (5x-x^2)-x\,dx = \int_0^4 4x-x^2\,dx$$
$$= 2x^2-\frac{x^3}{3}\Big|_0^4 = 2\cdot 16 - \frac{64}{3} = \frac{32}{3} = 10.\overline{66}$$

2.



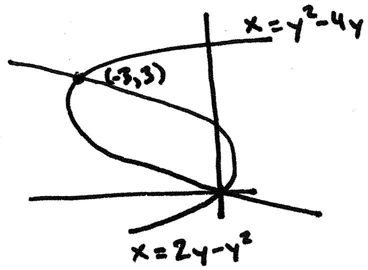
$$A = \int_0^2 \sqrt{x+2} - \frac{1}{x+1}\,dx = \int_0^2 \sqrt{x+2}\,dx - \int_0^2 (x+1)^{-1}\,dx$$
$$= \left[\frac{2}{3}(x+2)^{3/2} - \ln|x+1|\right]_0^2 = \left[\frac{2}{3}(4)^{3/2} - \ln 3\right] - \left[\frac{2}{3}(2)^{3/2} - \ln 1\right]$$
$$= \frac{16}{3} - \ln 3 - \frac{4\sqrt{2}}{3} = \frac{16-4\sqrt{2}}{3} - \ln 3 = 2.34910296$$

3.



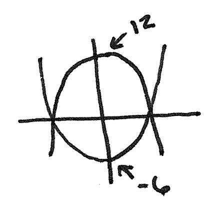
$$A = \int_{-1}^1 \left[e^y - (y^2-2)\right]dy = e^y - \frac{y^3}{3} + 2y\Big|_{-1}^1$$
$$= (e - 1/3 + 2) - (e^{-1} + 1/3 - 2) = e - e^{-1} + \frac{10}{3} = \frac{3e^2-3+10e}{3e}$$
$$= \frac{3e^2+10e-3}{3e} \qquad = 5.6837$$

4.



$$A = \int_0^3 (2y-y^2)-(y^2-4y)\,dy = \int_0^3 6y-2y^2\,dy$$
$$= 3y^2 - \frac{2y^3}{3}\Big|_0^3 = 27-18 = 9$$

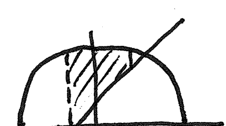
9. $y=12-x^2$, $y=x^2-6$



$$A = \int_{-3}^3 (12-x^2)-(x^2-6)\,dx$$
$$= \int_{-3}^3 18-2x^2\,dx$$
$$= 18x - \frac{2}{3}x^3\Big|_{-3}^3$$
$$= 36-(-36) = 72$$

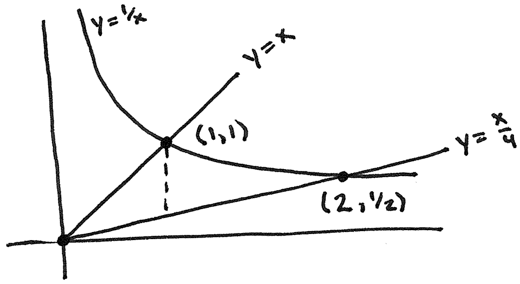
$12-x^2 = x^2-6$
$18 = 2x^2$
$9 = x^2$
$x = \pm 3$

5. $y=x+1$, $y=9-x^2$, $x=-1$, $x=2$



$$A = \int_{-1}^2 9-x^2-x-1\,dx$$
$$= \int_{-1}^2 8-x-x^2\,dx$$
$$= 8x - \frac{x^2}{2} - \frac{x^3}{3}\Big|_{-1}^2$$
$$= \left[16-2-8/3\right] - \left[-8-1/2+1/3\right] = \frac{39}{2} = 19.5$$

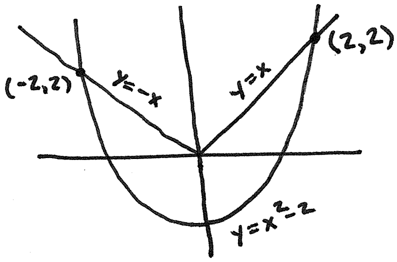
15. $y=1/x$, $y=x$, $y=1/4\,x$, $x\geq 0$



$\frac{1}{x} = \frac{x}{4}$
$x^2=4$
$x=2$

$$A = \int_0^1 x - \frac{x}{4}\,dx + \int_1^2 \frac{1}{x} - \frac{x}{4}\,dx$$
$$= \int_0^1 \frac{3}{4}x\,dx + \int_1^2 \frac{1}{x} - \frac{x}{4}\,dx$$
$$= \frac{3}{8}x^2\Big|_0^1 + \left[\ln x - \frac{x^2}{8}\right]_1^2$$
$$= 3/8 + (\ln 2 - 1/2) - (0 - 1/8)$$
$$= \ln 2 = .693147$$

16. $y = |x|$, $y = x^2 - 2$



$$A = \int_{-2}^{0} -x-(x^2-2)\,dx + \int_{0}^{2} x-(x^2-2)\,dx$$

equal

$$= 2\int_0^2 x - x^2 + 2\,dx$$
$$= 2\left[\frac{x^2}{2} - \frac{x^3}{3} + 2x\right]_0^2$$
$$= 2\left[2 - 8/3 + 4\right] = 20/3$$

24. $\Delta x = 20$, $n = 10$

$h_0 = 5.8$, $h_1 = 20.3$, . . . , $h_{10} = 2.8$

$$S_n = \frac{\Delta x}{3}\left[f(x_0) + 4f(x_1) + 2f(x_2) + \dots + 4f(x_{n-1}) + f(x_n)\right]$$

$$S = \frac{20}{3}\left[h_0 + 4h_1 + 2h_2 + \dots + 4h_9 + h_{10}\right]$$
$$= \frac{20}{3}\left[5.8 + 4(20.3) + 2(26.7) + 4(29) + 2(27.6) + 4(27.3) + 2(23.8) + 4(20.5) + 2(15.1) + 4(8.7) + 2.8\right]$$
$$= 4121.\overline{3}$$

38. $y = x^2$, $y =$ tangent line at $(1,1)$, $y = 0$

tangent line:

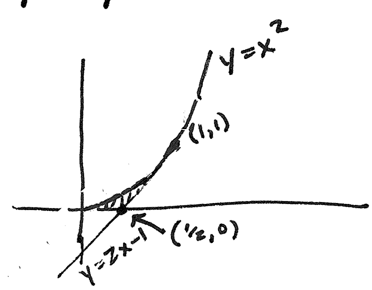
$y' = 2x$

$m = 2(1) = 2$

$y = 2x + b$

$1 = 2 + b$, $b = -1$

$y = 2x - 1$ is tangent line



$$A_1 = \int_{0.5}^{1} x^2 - (2x-1)\,dx$$
$$= \int_{0.5}^{1} x^2 - 2x + 1\,dx$$
$$= \frac{x^3}{3} - x^2 + x \Big|_{0.5}^{1}$$
$$= (1/3 - 1 + 1) - \left(\frac{(.5)^3}{3} - (.5)^2 + .5\right) = \frac{1}{24}$$

$$A_2 = \int_0^{1/2} x^2 - 0\,dx$$
$$= \frac{x^3}{3}\Big|_0^{1/2} = \frac{(1/2)^3}{3} = \frac{1}{24}$$

Area: $A_1 + A_2 = \frac{2}{24} = 1/12 = .08\overline{3}$